Inverse Gamma Distribution based Delay and Slew Modeling for On- Chip VLSI RC Interconnect for Arbitrary Ramp Input

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***Abstract*—The Elmore delay is fast becoming ineffective for deep submicron technologies, and reduced order transfer function delays are impractical for use as early-phase design metrics or as design optimization cost functions. This paper describes an accurate approach for fitting moments of the impulse response to probability density functions so that delay and slew metric can be estimated accurately at an early physical design stage. PERI (Probability distribution function Extension for Ramp Inputs) technique has been used that extends the delay and slew metrics for step inputs to the more general and realistic non-step or ramp inputs. The accuracy of the proposed model is justified by the results obtained from the proposed model and that of SPICE simulations.**

***Index Terms*—Moment Matching, Delay Calculation, Slew Calculation, Inverse Gamma Distribution, VLSI.**

I. Introduction

The advent of sub-quarter-micron IC technologies has forced dramatic changes in the design and manufacturing methodologies for integrated circuits and systems. The paradigm shift for interconnect which was once considered just a parasitic but can now be the dominant factor to determine the integrated circuit performances. It results the greatest impetus for the change in existing methodologies. As integrated circuit feature sizes continue to scale well below 0.18 microns, active device counts are reaching hundreds of millions [3]. This paper proposes an extension of Elmore's approximation [1] to include matching of higher order moments of the probability density function. Specifically, using a time-shifted incomplete gamma function approximation [2] for the impulse responses of RC trees, the three parameters of this model are fitted by matching the first three central moments (mean, variance, skewness), which is equivalent to matching the first two moments of the circuit response (m_1, m_2,). Importantly, it is proved that such a gamma fit is guaranteed to be realizable and stable for the moments of an RC tree [4]. In this work, we used PERI technique [13] for extending the delay metric derived for a step input into a delay metric for a ramp input for RC trees that is valid over all input slews. A noteworthy feature of this method is that the delay metric reduces to the Elmore delay of the circuit under the limiting case of an infinitely slow ramp, a fact first proved in [6] to establish the Elmore delay as an upper bound.

The remainder of the paper is organized as follows: Section 2 presents the pertinent background information relating to the probability and circuit theory. Section 3 shows the proposed techniques for extending step delay and slew metrics to ramp inputs. Section 4 presents the simulation results that validate the effectiveness of the approach. Finally, Section 5 concludes the paper.

II. Basic Theory

A. Moments of a Linear Circuit Response

Let h(t) be a circuit impulse response in the time domain and let H(s) be the corresponding transfer function. By definition, H(s) is the Laplace transform of h(t) [12],

$$H(s)=\int_0^\infty h(t)e^{-st}\,dt \tag{1}$$

Applying the Taylor series expansion of e^{-st} about s=0 yields,

$$H(s)=\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}s^i\int_0^\infty t^i h(t)dt \tag{2}$$

The i[th] circuit-response moment, $\tilde{m}_i$ is defined as [12]:

$$\tilde{m}_i=\frac{(-1)^i}{i!}\int_0^\infty t^i h(t)dt \tag{3}$$

From (2) and (3), the transfer function H(s) can be expressed as:

$$H(s)=\tilde{m}_0+\tilde{m}_1 s+\tilde{m}_2 s^2+\tilde{m}_3 s^3+\dots \tag{4}$$

B. Central Moments

It is straightforward to show that the first few central moments can be expressed in terms of circuit moments as follows [6]:

$$\mu_0=m_0,\mu_1=0,\mu_2=2m_2-\frac{m_1^2}{m_0},\mu_3=-6m_3+6\frac{m_1m_2}{m_0}-2\frac{m_1^3}{m_0^2} \tag{5}$$

Unlike the moments of the impulse response, the central moments have the geometrical interpretations:

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μ_0 is the area under the curve. It is generally unity, or else a simple scaling factor is applied. μ_2 is the variance of the distribution which measures the spread or the dispersion of the curve from the center. A larger variance reflects a larger spread of the curve. μ_3 is a measure of the skewness of the distribution.

III. Proposed Work

A. Calculation of the Delay Metric

Elmore's distribution interpretation can be extended beyond simply estimating the median by the mean if higher order moments can be used to characterize a representative distribution function. Once characterized, the delay can be approximated via closed form expression or table lookup of the median value for the representative distribution family. The inverse gamma distribution is a two parameter continuous distribution [4]. The PDF of Inverse Gamma's distribution [4] is depicted in Figure 1. The inverse gamma distribution's probability density function is defined over x > 0,

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\left(\frac{1}{x}\right)^{\alpha+1}e^{-\frac{\beta}{x}} \tag{12}$$

Where α is the shape parameter and β is the scale parameter. Mean for the inverse gamma distribution is given by,

$$Mean(E[x])=\frac{\beta}{\alpha-1} \tag{13}$$

Variance of the inverse gamma distribution is given by,

$$Variance=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)} \tag{14}$$

Mode of the inverse gamma distribution is given by,

$$Mode=\frac{\beta}{\alpha+1} \tag{15}$$

The parameters α and β of inverse gamma distribution can be presented in terms of moments. Moment generating function of inverse gamma distribution is given as,

$$E\left[X^n\right]=\frac{\beta^n}{(\alpha-1)(\alpha-2)..(\alpha-n)} \tag{16}$$

When, n=1; $E[X]=\frac{\beta}{\alpha-1}$ (17)

When, n=2; $E\left[X^2\right]=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$ (18)

Hence we can write,

$$m_1=\frac{\beta}{\alpha-1} \tag{19}$$

$$m_2=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)} \tag{20}$$

From (19) and (20),

$$\left(\alpha^2-3\alpha+2\right)m_2=(\alpha-1)^2 m_1^2 \tag{21}$$

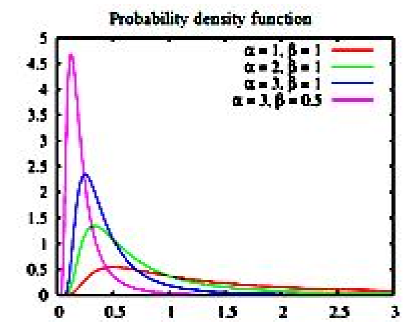

Figure 1. Inverse Gamma Distribution PDF.

After solving above equation,

$$\alpha^2\left(m_2-m_1^2\right)+\alpha\left(2m_1^2-3m_2\right)+\left(m_2-m_1^2\right)=0 \tag{22}$$

By solving (22), we get,

$$\alpha=\frac{-\left(2m_1^2-3m_2\right)\pm\sqrt{\left(2m_1^2-3m_2\right)^2-4\left(m_2-m_1^2\right)\left(2m_2-m_1^2\right)}}{4\left(m_2-m_1^2\right)} \tag{23}$$

$$\alpha=\frac{-\left(2m_1^2-3m_2\right)\pm\sqrt{\left(4m_1^4+9m_2^2-12m_1^2m_2-8m_2^2+8m_1^2m_2+4m_1^2m_2-4m_1^4\right)}}{4\left(m_2-m_1^2\right)}$$

By taking positive sign, we get for α=1/2,

$$\alpha=\frac{4m_2-2m_1^2}{4\left(m_2-m_1^2\right)} \tag{24}$$

From (19) and (24), we can write for β as,

$$\beta=\frac{2m_1^3}{4\left(m_2-m_1^2\right)} \tag{25}$$

The median of the inverse gamma distribution is defined as [9],

$$Median=\frac{1}{3}\left[Mode+2Mean\right] \tag{26}$$

Substituting (13) and (15), we get,

$$Median=\frac{1}{3}\left[\frac{2\beta}{\alpha-1}+\frac{\beta}{\alpha+1}\right] \tag{27}$$

After solving,

$$Median=\frac{(3\alpha+1)\beta}{3(\alpha+1)(\alpha-1)} \tag{28}$$

Now substituting the values of α and β from (24) and (25) in (28), we have,

$$Median=\frac{\left(3\frac{2\left(m_2-m_1^2\right)}{4\left(m_2-m_1^2\right)}+1\right)\frac{2m_1^3}{4\left(m_2-m_1^2\right)}}{3\left(\frac{2\left(m_2-m_1^2\right)}{4\left(m_2-m_1^2\right)}+1\right)\left(\frac{2\left(m_2-m_1^2\right)}{4\left(m_2-m_1^2\right)}-1\right)} \tag{29}$$

After simplification of the above equation, we get,

$$50\%\ Delay=Median=\frac{m_1\left(8m_2-5m_1^2\right)}{3\left(4m_2-3m_1^2\right)} \tag{30}$$

The above equation (30) is the proposed closed form delay expression for generalized RC global interconnects using inverse gamma distribution based on moment generating technique.

B. Calculation for the Slew Metric

The step response of an RC circuit is a cumulative density function (CDF) [7]. The RC response is considered as a single-

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pole exponential waveform and can be modeled as,

$h(t) = 1 - e^{-\frac{t}{\beta}}$, $t > 0$, If h(t) satisfies the following conditions:

$$0 \le h(t) \le 1 \text{ and } \lim_{t \to -\infty} f(t) = 0, \lim_{t \to \infty} f(t) = 1 \quad (31)$$

Now, let T_{LO} and T_{HI} be 10% and 90% delay points, respectively. Matching to these points to the CDF yields,

$$0.1 = 1 - e^{-\frac{T_{LO}}{\beta}} \quad (32)$$

$$0.9 = 1 - e^{-\frac{T_{HI}}{\beta}} \quad (33)$$

From (32) and (33) we have,

$$T_{LO} = \lambda \ln\left(\frac{10}{9}\right) = 0.1053\beta \quad (34)$$

$$T_{HI} = \lambda \ln(10) = 2.302\beta \quad (35)$$

Using (34) and (35), we define the inverse gamma slew metric (we call this metric as IGSM) as,

$$IGSM = T_{HI} - T_{LO} = 2.1976\beta \quad (36)$$

Using equation (24) and (25), we can write the closed form expression of the slew metric in terms of first two circuit moments as

$$IGSM = \frac{4.3952 m_1^3}{4\left(m_2 - m_1^2\right)} \quad (37)$$

From the above derived equation (37) for the slew metric for the on-chip interconnect using inverse gamma distribution function.

C. Delay Metric for Ramp Input

Let $\mu(s) = -m_1$ is the Elmore delay and M(S) is the step delay metric as given (30). The delay estimation for the ramp response [13] is given by,

$$D(R) = (1-\alpha)\,\mu(s) + \alpha\, M(S) \quad (38)$$

Where α denotes the constant and is given as,

$$\alpha = \left(\frac{2m_2 - m_1^2}{2m_2 - m_1^2 + \frac{T^2}{12}}\right)^{\frac{5}{2}} \quad (39)$$

Where 0<T<“ is the slope of the ramp input.

From (30) and (38), we get,

$$D(R) = -m_1 + \alpha\left(\frac{2m_2}{3m_1} - \frac{m_1}{3}\right) \quad (40)$$

Substituting the value of α from (39) in (40) we get,

$$D(R) = -m_1 - \left(\frac{2m_2}{3m_1} - \frac{m_1}{3}\right)\left(\frac{2m_2 - m_1^2}{2m_2 - m_1^2 + \frac{T^2}{12}}\right)^{\frac{5}{2}} \quad (41)$$

Equation (41) is the delay metric using inverse gamma distribution function for ramp input.

D. Slew Metric for Ramp Input

The output slew is the root-mean square of the step slew and input slew [14]. For ramp slew

$$Slew(R) = \sqrt{Slew^2(S) + Slew^2(I)} \quad (42)$$

Further, (42) exhibits the right limiting behavior: as $Slew(I) \to \infty$, we have $Slew(R) \to \infty$ and as $Slew(I) \to 0$, we have $Slew(R) \to Slew(S)$. Where Slew(S) is the step slew metric which is given by (37) and Slew (I) is the input slew which is given as,

$$Slew^2(I) = T^2/12 \quad (43)$$

From (37), (42) and (43), we get

$$Slew(R) = \sqrt{\left(\frac{4.3952 m_1^3}{4\left(m_2 - m_1^2\right)}\right)^2 + \frac{T^2}{12}} \quad (44)$$

The above equation (44) is the slew metric equation for the inverse gamma distribution function for ramp input.

IV. Simulation Results and Discussions

We have implemented the proposed delay estimation method using inverse gamma distribution and applied it to widely used actual interconnect RC networks as shown in Figure-2. For each RC network source we put a driver, where the driver is a voltage source followed by a resister. We classify the 2224 sinks as it was taken in PERI [14]

- 1187 far-end sinks have delay greater or equal to 75% of the maximum delay to the furthest sink in the net.
- 670 mid-end sinks which have delay between 25% and 75% of the maximum delay and,
- 367 near-end sinks which have delay less than or equal to 25% of the maximum delay.

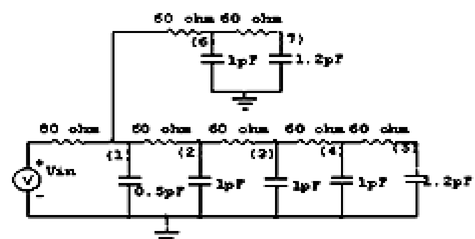

Figure 2. An RC Tree Example

We compare the obtained average, minimum, maximum values along with standard deviation for delay from PERI with those found using our proposed model. The comparative results are summarized in Table 1.

Table I.
Comparison of the Proposed Delay Model with Peri

Delay	Delay Metric Using Two Moments							
Ramp	PERI Method				Our Proposed Model			
Sinks	Avg	SD	Max	min	Avg	SD	max	Min
Near	1.25	0.33	2.25	0.50	1.24	0.41	2.24	0.48
Mid	1.14	0.09	1.47	0.97	1.09	0.10	1.37	0.91
Far	1.00	0.01	1.03	0.98	1.12	0.97	1.14	0.92
Total	1.08	0.17	2.25	0.50	1.01	0.13	0.54	0.51

Table I.
Comparison of the Proposed Slew Model with Peri

Slew	Slew Metric Using Two Moments							
Ramp	PERI Method				Our Proposed Model			
Sinks	Avg	SD	Max	min	Avg	SD	max	min
Near	1.01	0.21	1.89	0.65	1.01	0.12	1.88	0.61
Mid	0.89	0.07	1.21	0.75	0.91	0.03	1.26	0.74
Far	1.13	0.06	1.25	0.98	1.11	0.04	1.11	0.73
Total	1.04	0.15	1.89	0.66	1.09	0.21	1.82	0.58

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We compare the obtained average, minimum, maximum values along with standard deviation for slew from PERI with those found using our proposed model. The comparative results are summarized in Table 2. For the final step for experiments, we have estimated the effect of input slew on the delay and slew estimation for the same seven node RC network as shown in the figure 3.we have chosen 10/90 input slew values of 200 and 500 picoseconds and delays and slews were estimated for each node. The comparative result of our proposed model with RICE [14] are shown in the Table 3 and Table4 for delay and slew, respectively.

TABLE III. COMPARISON OF THE PROPOSED SLEW MODEL WITH PERI

Input Slew	250 ps		500 ps	
Node	RICE	Proposed Model	RICE	Proposed Model
1	1659	1657	1758	1728
2	1733	1739	1816	1810
3	2003	1997	2079	2072
4	2164	2162	2219	2221
5	2223	2231	2261	2279
6	1748	1742	1826	1819
7	2233	2237	2276	2274

TABLE I.
COMPARISON OF THE PROPOSED SLEW MODEL WITH PERI

Input Slew				
Node	RICE	Proposed Model	RICE	Proposed Model
1	210	209	272	270
2	383	381	409	406
3	482	480	498	502
4	705	707	716	729
5	849	855	859	859
6	461	462	487	483
7	923	933	933	939

V. CONCLUSIONS

In this paper, we have extended the method to estimate the interconnect delay metric for high speed VLSI designs from step input to ramp input. We have used inverse gamma probability distribution function to derive our metric. Our model has Elmore delay as upper bound but with significantly less error. The novelty of our approach is justified by the comparison made with that of the results obtained by SPICE simulations.

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